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THE THEORY OF ELECTRICITY

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THIS VOLUME IS IN GRATITUDE DEDICATED TO
THE MASTER AND FELLOWS OF JESUS COLLEGE
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PREFACE

THE following work is offered as a general text book on the mathematical aspects of modern electrical theory, and incidentally also as an attempt to present the complete subject in a consistent form. There seems for a comprehensive work of this kind, for in the standard text book on this subject the treatment, besides being incomplete, is often far from convincing and at times not free from error.

The treatment is based mainly on the original Faraday-Maxwell theory generalised and extended to the case of moving systems by Sir Joseph Larmor. This form of the theory has been almost entirely abandoned in recent accounts of the subject, but it remains the only one which appears to be completely satisfactory from the point of view of mathematical and physical consistency, and in its generality it is unapproached by any other form.

Although the present exposition is essentially a mathematical one, much of the purely analytical mathematics usually associated with the subject has been omitted. Particular attention has however been given to the rigorous formulation of underlying physical principles and to their translation into a mathematical theory. The dynamical aspects of the subject have been specially emphasised throughout.

In the development of the general plan and of the details of the book I have derived great assistance from my notes of lectures delivered by Sir Joseph Larmor at Cambridge during the academical year 1909-10, afterwards supplemented from his various published works, more particularly the papers, 'On a dynamical theory of the electric and luminiferous medium' [*Phil. Trans.* 1894-1897] and the book, *Aether and Matter* [Cambridge, 1900]. I am extremely grateful to Professor Larmor for his kind permission to make free use of these notes.

CORRECTIONS AND ADDITIONS.

- pp. 39, 158, 661. Until quite recently I have not had an opportunity of examining Einstein's generalised theory of relativity and was in consequence unable to take up any other point of view as regards gravitational phenomena.
- p. 84, § 88. The argument of the exponential function under the integral sign in the formulae for ϕ should read $(-kz)$ not $(-kr)$.
- p. 338. The values for the electronic charge given on this page are those first obtained by the method under discussion: subsequent improvements and corrections have increased this value by about 50 % to that given on p. 341 and in other parts of the work.
- p. 345, § 388. In the expression for the potential of a magnetic shell the solid angle is measured by that area of the unit sphere round the field point which is such that if it were coated with a double sheet with a similar disposition to that on the given shell, then the direction of the outward drawn radius at any point would correspond to the positive direction of magnetisation in this sheet.

to the sense in which the first integral is taken round the circuit in the same manner as translation to rotation in a left-handed screw relation.

In order to prove this theorem it is first necessary to notice that it will be merely sufficient to prove it for a small elementary plane circuit; because if we consider any finite barrier surface divided up into a large number of such small surfaces by means of a network of lines, then the total sum of the line integrals taken for each little elementary circuit separately is equivalent to the integral taken round the outer circuit alone, any interior element of a circuit being counted on the whole twice over with opposite signs in each case.

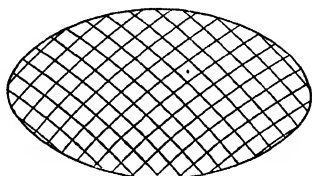


Fig. 5

We shall therefore content ourselves with proving the theorem for a small plane area.

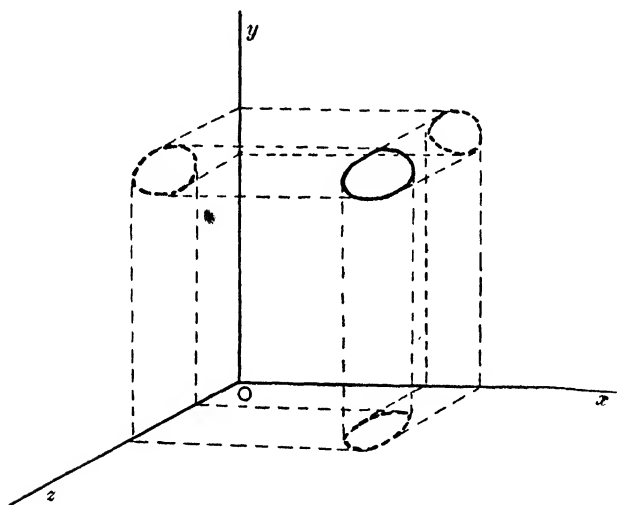


Fig. 6

Consider the integral

$$\int \mathbf{A}_x dx$$

taken round the boundary of such an element. If \mathbf{A}_x is the value of this quantity at the mean centre of the element, the value at a point on the boundary whose small coordinates relative to this centre are (x', y', z') is

$$\mathbf{A}_x + x' \frac{\partial \mathbf{A}_x}{\partial x} + y' \frac{\partial \mathbf{A}_x}{\partial y} + z' \frac{\partial \mathbf{A}_x}{\partial z},$$

discovered by Thomson, but experimental evidence is gradually tending to the view that the positively charged hydrogen atom is the ultimate element of the positive electricity which exists in all substances. The element of positive electricity if assumed to have the mass of a hydrogen atom also carries the same charge, viz. $3 \cdot 10^{-10}$ units; but its mass is 1700 times that of an electron.

46. There is now an enormous mass of experimental evidence, to which contributions are made, not only by the phenomena of electrostatics, but also by the phenomena of almost every branch of physics and chemistry tending to show that each chemical atom of matter contains as an essential part of its constitution a certain number of electrons grouped together in various more or less stable congeries; each atom also possesses in some as yet undetermined form the necessary positive electric charge to make it electrically neutral on the whole. In every solid body there is a continual process of atomic dissociation going on, the electronic configuration inside the atom being sufficiently unstable in many cases to be capable of breaking up on small provocation, with the consequent liberation of one or more electrons and occasionally of positive elements as well: the result of this is that mixed up with the atoms of chemical matter composing a body we have a greater or less percentage of negative electrons, and a few positive elements freely moveable in the interstices between the atoms. It is in fact to these free electrons and positive charges that the phenomena of electric conduction is due. An electrically charged body is one in which there is an excess or deficit of (negative) electrons. The action between the charges of the electrons and the charges in the atoms is precisely that specified by Coulomb's law provided the charges are at rest and at distances from one another large compared with ordinary molecular dimensions. The distinction between insulators and conductors as regards the phenomena of induction and conduction depends essentially on the fact that in the conductors there is a large number of the free dissociated electrons which can be pulled about from one part of the medium to another under the action of forces from other electrified bodies; whereas in insulators there is such an extremely small number of these free electrons, that the phenomena depending on them can under most circumstances be neglected.

47. We shall in our future investigations discuss many facts which have led up to this conception of the essential electronic constituent of matter; but we may here just mention one important point in its favour to which we shall not have any need to refer to in our future work. In 1896 Becquerel discovered the so-called radioactive substances, which are continually and spontaneously emitting a complicated type of radiation, of which two of the main constituents have been proved to be composed, the one of rapidly moving electrons (β particles) and the other of more slowly moving positive

CHAPTER. II

THE CHARACTERISTIC PROPERTIES OF THE ELECTRIC FIELD

72. Some particular types of electric fields. Having now obtained consistent definitions of the analytical functions determining the field of any distribution of electric charges we may proceed to examine the properties of these functions which are characteristic of the fields to which they appertain. Before however entering on the general examination it seems desirable to consider the form which the definitions assume in the case of certain simple specified distributions, with the view principally to obtaining some insight into the analytical nature of the functions involved. Most distributions of charge may be regarded as more or less approximately composed of a certain number of simple distributions of standard type, so that if we know the nature of the fields associated with these simple distributions we shall be in a position to obtain an approximate estimate at least of the nature of any more complex distribution.

It is of course not necessary in each case to determine all the integrals discussed in the last chapter. When the integral for the potential is known in any case the components of force in any direction may be most simply derived as the component of the vector

$$\mathbf{E} = - \nabla \phi$$

or as the negative gradient of the potential in that direction, and it is not necessary to evaluate separately the integrals for the force components. We shall therefore merely discuss the integral for the potential function in the separate cases.

73. The potential of a linear distribution of charge. The charge is continuously distributed along a line of continuous curvature. The charge on the element of length ds is dq and

$$v = \frac{dq}{ds}$$

is the line density; the conception of which depends on the existence or the differential coefficient expressing it. Physically the charge is so concentrated around a line that for all practical purposes it is convenient to regard it as actually on the line. The potential function is

$$\phi = \int \frac{dq}{r} = \int \frac{v ds}{r}.$$

(c) The uniformly charged conducting sphere.

The method is the same : apply Gauss' theorem to concentric spherical surfaces.

(d) We can determine in the same way the field of force for two concentric spherical conductors (radii a_1 and a_2) carrying charges Q_1 and Q_2 and prove that if $Q_1 = -Q_2$ the field is confined to the space between the conductors.

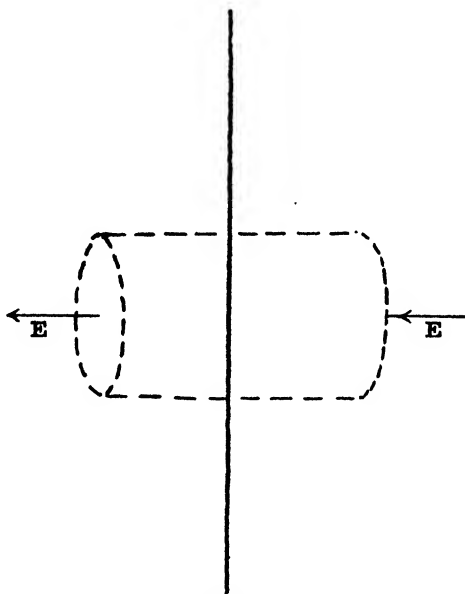


Fig. 13

The force inside the inner sphere is zero : in the space between the spheres it is radial and symmetrical and at a distance r from the centre it amounts to

$$\frac{Q_1}{r^2}.$$

Outside both spheres the force is also symmetrical and radial but is now of intensity

$$\frac{Q_1 + Q_2}{r^2},$$

at a distance r from the centre.

93. We now make use of the general theorem to analyse the properties of electrostatic fields in such a way as will lead most directly to a consideration of Faraday's speculations on the origin of all electrical actions; but before going on to this it is interesting to notice that when we consider

